Learn at Home with Diverse Online Resources

In view of the impact brought by the new coronavirus, schools in Hong Kong have delayed school resumption after the Lunar New Year Holiday. Let's make good use of this online resources to facilitate us to learn at home.

Name: _______________(　　)

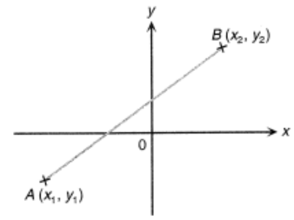
Learn at Home F4 Math worksheet 15.1 Junior form Coordinate Geometry

Key Points

Distance Formula

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ on a rectangular coordinate plane is given by:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Slopes of Straight Lines

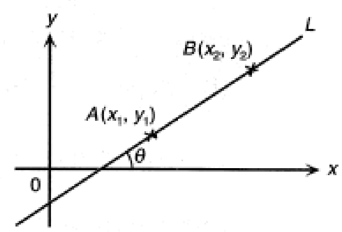
1. Slope and inclination of a straight line
Consider a straight line L passing through $A(x_1, y_1)$ and $B(x_2, y_2)$.

    (a) The slope m of L is given by:

    $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

    (b) The inclination θ of L is the angle that L makes with the positive x-axis, where $0° \le \theta \le 180°$.

    $$\tan\theta = m = \frac{y_2 - y_1}{x_2 - x_1}$$



2. Parallel lines and perpendicular lines
Let m_1 and m_2 be the slopes of straight lines L_1 and L_2 respectively.

    (a) If $L_1 // L_2$, then $m_1 = m_2$.
    Conversely, if $m_1 = m_2$, then $L_1 // L_2$.

    (b) If $L_1 \perp L_2$, then $m_1 \times m_2 = -1$.
    Conversely, if $m_1 \times m_2 = -1$, then $L_1 \perp L_2$.

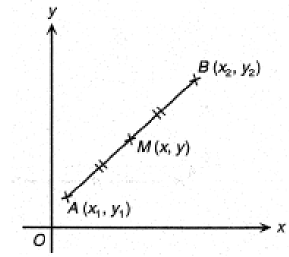
3. Collinear points

    Consider three points A, B and C. If line segments AB and BC with the common end point B, and AB and BC have the same slope, then points A, B and C are collinear.

Mid-point Formula

If $M(x, y)$ is the mid-point of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$, then

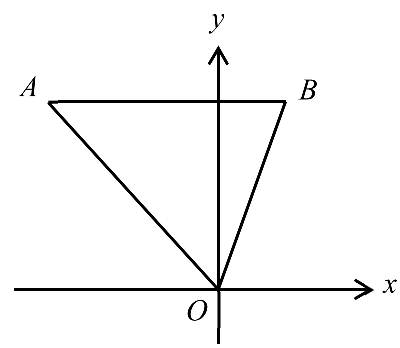
$$x = \frac{x_1 + x_2}{2} \text{ and } y = \frac{y_1 + y_2}{2}.$$



1. In the figure, O, $A(a, 12)$ and $B(5, 12)$ are the vertices of a triangle.

(a) Express the lengths of OA and AB in terms of a.

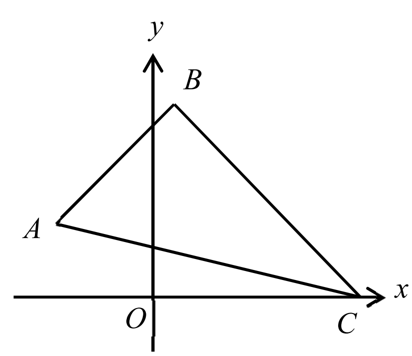
(b) If the perimeter of $\triangle AOB$ is 42 units, find the value of a.



2. In the figure, $A(-4, 4)$, $B(1, 9)$ and $C(10, 0)$ are the vertices of a triangle.

(a) Show that $\angle ABC = 90°$.

(b) Hence, find the area of $\triangle ABC$.



3. $A(0, 6)$, $B(-2, m)$, $C(4, 2)$ and $D(6, 7)$ are the vertices of a parallelogram.

(a) Find the value of m.

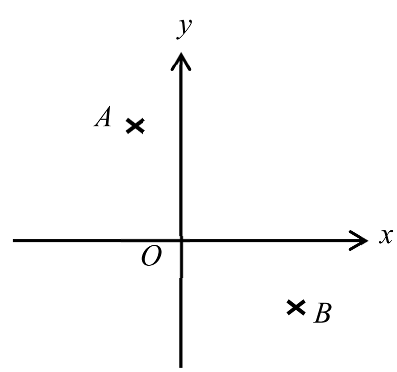
(b) Lena claims that no points inside the parallelogram lie in quadrant IV. Do you agree? Explain your answer.

4. Given that $A(5, -2)$, $B(-1, k)$ and $C(-4, 1)$ are collinear points on a rectangular coordinate plane.

(a) Find the value of k.

(b) Find the mid-point of AB.

5. The figure shows a triangle with vertices $A(-6, h)$, $B(-8, 1)$ and $C(k, 1)$. It is given that $AC = BC$ and the slope of AB is 3.



(a) Find the values of h and k.

(b) **(i)** Find the area of $\triangle ABC$.

(ii) D is a point on AB such that $CD \perp AB$.

Show that the length of CD is $3\sqrt{10}$ units.

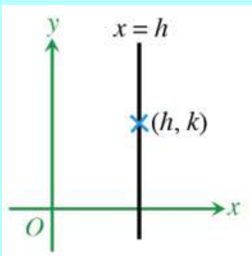
Learn at Home F4 Math worksheet 15.2 Vertical line and Horizontal line

Before finish the following excise, let's have a Math chat by scanning the QR code

https://qrgo.page.link/1A5L1

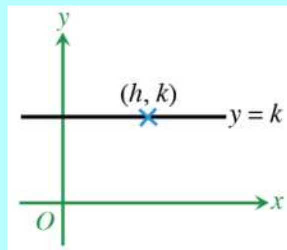
Vertical Lines

The equation of a vertical line passing through point (h, k) is $x = h$.



Horizontal Lines

The equation of a horizontal line passing through point (h, k) is $y = k$.



1. Find the equation of the straight line which passes through $(-2, -5)$ and is parallel to the x-axis.

→Exercise 15.1: 4 – 5

2. Find the equation of the straight line which passes through $(8, -3)$ and has a x-intercept 8.

→Exercise 15.1: 28

3. Consider three straight lines L_1: $y = 9$, L_2: $x = -6$ and L_3: $x + 3 = 0$.

(a) Which straight lines are parallel to each other?

(b) Which straight lines are perpendicular to each other?

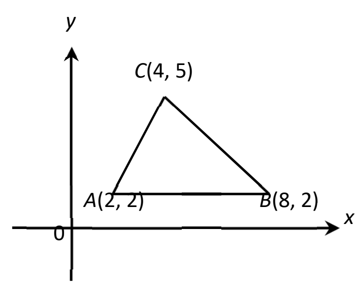
4. Find the equation(s) of the straight line which has a distance of 3 units from the line $x = 2$.

5. Find the equation(s) of the straight line which makes an angle of 45° with a straight line L and passes through (2, 3). The slope of L is –1 which also passes through (2, 3).

6. In the figure, the coordinates of A, B and C are (2, 2), (8, 2) and (4, 5) respectively.

(a) Find the equation of the perpendicular bisector of AB.

(b) Hence, find the x-coordinate of the circumcentre of $\triangle ABC$.



Learn at Home F4 Math worksheet 15.3 Point-slope Form

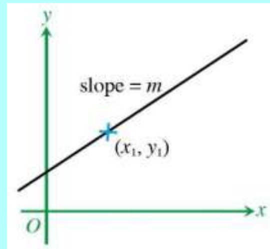
Slope m of a line joining any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2$$

Point-slope Form

The equation of a straight line passing through (x_1, y_1) with slope m is given by:

$$y - y_1 = m(x - x_1)$$

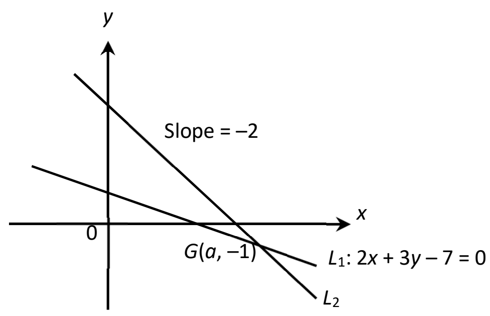


1. Find the equation of the straight line passing through $A(-2, 4)$ and $B(3, -7)$.

2. In the figure, $L_1: 2x + 3y - 7 = 0$ is a straight line and L_2 is another straight line with slope -2. L_1 and L_2 intersect at $G(a, -1)$.

(a) Find the value of a.

(b) Find the equation of L_2.



→Exercise 15.1: 11 – 12

3. A straight line L with inclination 30° passes through the point (0, –3).

(a) Find the equation of L. →Exercise 15.1: 26

(*Leave the answer in surd form*.)

(b) Determine whether L passes through ($\sqrt{12}$, –1).

4. The coordinates of E and F are (–4, –3) and (2, –1) respectively. Find the equation of the perpendicular bisector of EF.

5. Q(–2, 2) is a point on PR with PQ : QR = 3 : 2. The coordinates of P are (–8, –1).

(a) Find the coordinates of R.

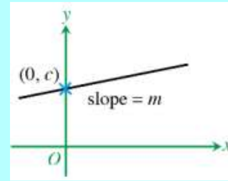
(b) Find the equation of the straight line L passing through R and S(5, –5).

Learn at Home F4 Math worksheet 15.4   Slope-intercept Form

Slope-intercept Form

The equation of a straight line with slope *m* and *y*-intercept *c* is given by:

$$y = mx + c$$

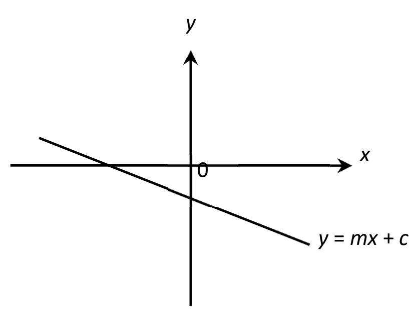


1. A straight line *L* has *x*-intercept $\frac{1}{2}$ and *y*-intercept –2.

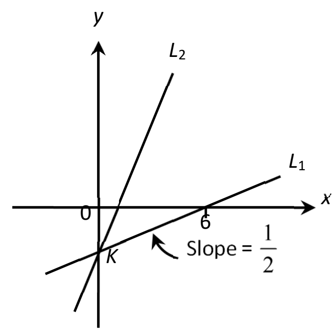
(a) Find the equation of *L*. →Exercise 15.1: 15 – 16

(b) Determine whether *L* passes through *A*(3, 10).

2. The figure shows the graph of $y = mx + c$. Determine whether *mc* is a positive number or a negative number.



3. In the figure, the slope and the x-intercept of L_1 are $\frac{1}{2}$ and 6 respectively. The slope of L_2 is triple that of L_1. Both lines intersect the y-axis at K.



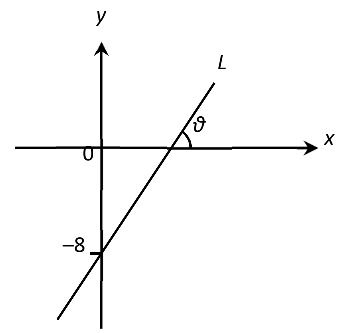
(a) Find the y-intercept of L_1.

(b) Find the equation of L_2.

4. L_1: $x - y + 3 = 0$ is a straight line. L_2 is another straight line such that L_1 and L_2 have the same y-intercept and $L_1 \perp L_2$. Find the equation of L_2.

5. In the figure, the inclination and the y-intercept of a straight line L are ϑ and -8 respectively.

It is given that $\sin\vartheta = \dfrac{8}{\sqrt{73}}$.



(a) Find the slope of L.

(b) L_1 is another straight line which is parallel to L. L_1 has a y-intercept half of that of L. Find the equation of L_1.

6. Let $ax + by + c = 0$ be the equation of a straight line L.

(a) Express the equation of L in the slope-intercept form. Hence, express the slope and the y-intercept of L in terms of a, b and c.

(b) If $2a = 5b = c$, find the slope and the y-intercept of L.

Learn at Home F4 Math worksheet 15.5 Slope and Intercept

Before finish the following excise, let's have a Math chat by scanning the QR code

https://qrgo.page.link/jYraB

General Form of the Equation of a Straight Line

$$Ax + By + C = 0,$$

where A, B and C are constants with A and B not both zero.

Consider the equation of a straight line L: $Ax + By + C = 0$, where $A \neq 0$ and $B \neq 0$.

x-intercept $= -\dfrac{C}{A}$ $\qquad$ y-intercept $= -\dfrac{C}{B}$ $\qquad$ Slope $= -\dfrac{A}{B}$

Note: **(i)** If $A = 0$, L is a horizontal line.

(ii) If $B = 0$, L is a vertical line.

1. In each of the following, find the slope, the x-intercept and the y-intercept of the given straight line.

(a) $-\dfrac{x}{9} + 7 = -y$ $\qquad$ **(b)** $\dfrac{1}{2}y = 4x - \dfrac{1}{3}$

→Exercise 15.2: 5 – 10

2. In each of the following, determine whether the straight lines L_1 and L_2 are parallel to each other, perpendicular to each other or neither of them.

(a) L_1: $2x - 3y + 1 = 0$; L_2: $6y - 4x - 3 = 0$

(b) L_1: $3x = 10 - 2y$; L_2: $6x - 9y = 5$

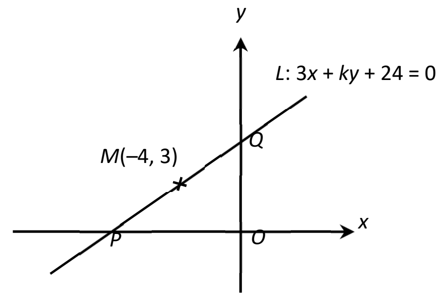
(c) L_1: $\dfrac{x}{2} - \dfrac{y}{5} = 1$; L_2: $4y - \dfrac{1}{6} = -2x$

→Exercise 15.2: 11 – 12

3. In the figure, the straight line L: $3x + ky + 24 = 0$ cuts the x-axis and the y-axis at P and Q respectively. L passes through $M(-4, 3)$.

(a) Find the coordinates of P and Q.

(b) Billy claims that the area of $\triangle POQ$ is greater than 20 square units. Do you agree? Explain your answer.



HKDSE Corner

4. In the figure, the equations of the straight line L_1 and L_2 are $ax = 1$ and $bx + cy = 1$ respectively.

Which of the following are true?

I. $a > 0$

II. $a > b$

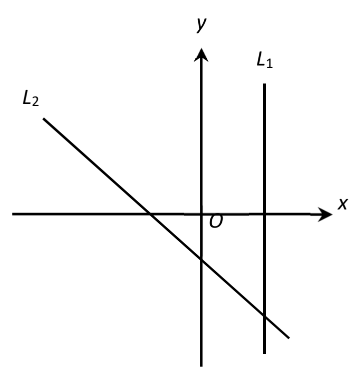
III. $c < 0$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III



5. The slope, the x-intercept and the y-intercept of a straight line are m, h and k respectively.

Prove that $k = -mh$.

Learn at Home F4 Math worksheet 15.6 Parallel lines and Perpendicular lines.

In this worksheet, give the answers for equations of straight lines in the general form.

1. L_1: $(k + 3)x + 2y + 7 = 0$ and L_2: $(k - 2)x - 3y - 3 = 0$ are two perpendicular straight lines.

Find the value of k.

2. Consider three points $P(-4, -1)$, $Q(1, 1)$ and $R(11, 5)$ on a rectangular coordinate plane.

(a) By finding the slopes of PQ and QR, determine whether P, Q and R are collinear.

(b) $S(13, -2)$ is a point on the same coordinate plane. Find the equation of the altitude from S to PR of $\triangle PRS$.

→Exercise 15.2: 30

3. $A(-3, 1)$, $B(1, 5)$ and $C(5, -3)$ are three vertices of $\triangle ABC$.

(a) Find the equations of the median from B to AC and the median from C to AB.

(b) Determine whether $T(1, 1)$ is the centriod of $\triangle ABC$.

→Exercise 15.2: 27

4. Consider the following two straight lines:

L_1: $4x + ky + 4 = 0$

L_2: $y = \dfrac{k}{4}x + 5$ $\quad\quad (k \neq 0)$

L_3 is another straight line which parallel to L_1 and has the same y-intercept as L_2.

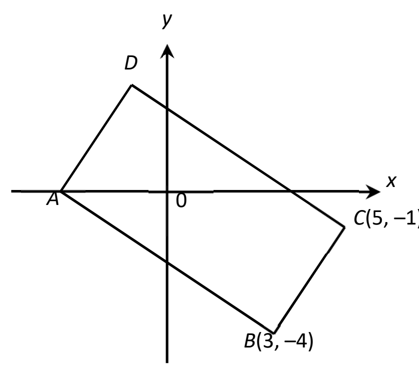
(a) Prove that $L_2 \perp L_3$.

(b) Find the equation of L_3 in terms of k.

5. In the figure, $ABCD$ is a rectangle. A is a point on the x-axis. The coordinates of B and C are (3, –4) and (5, –1) respectively.

(a) Find the coordinates of A.

(b) Find the equation of AD.



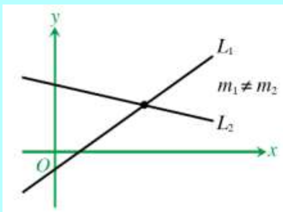
Learn at Home F4 Math worksheet 15.7 No. of points of intersection of 2 st. lines

Before finish the following excise, let's have a Math chat by scanning the QR code

https://qrgo.page.link/YQzzn

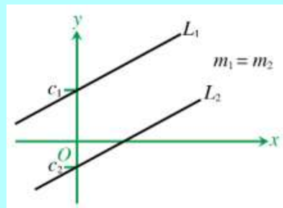
A. Number of Points of Intersection of Two Straight Lines L_1 and L_2:

(i) If $m_1 \neq m_2$,



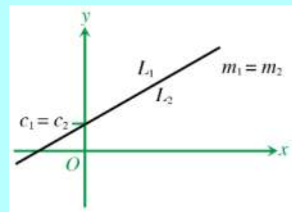
there is only one point of intersection.

(ii) If $m_1 = m_2$ and $c_1 \neq c_2$,



there are no points of intersection.
(L_1 // L_2)

(iii) If $m_1 = m_2$ and $c_1 = c_2$,



there are infinitely many points of intersection.
(L_2 overlaps with L_1.)

B. Finding the Number of Points of Intersection of L_1 and L_2:

Step 1: check whether $m_1 = m_2$.

Step 2: check whether $c_1 = c_2$.

1. In each of the following, find the number of points of intersection of L_1 and L_2.

(a) L_1: $x - y + 8 = 0$, L_2: $3y = 9 - 2x$

(b) L_1: $5y = 3x$, L_2: $6x = 1 + 10y$

→Exercise 15.3: 1 – 6, 13

(c) L_1: $y = 4x - 3$, L_2: $8 = x - 8y$

(d) L_1: $y = -\frac{2}{3}x - 8$, L_2: $\frac{x}{3} + \frac{y}{2} = -4$

2. Two straight lines L_1: $5 = kx - 5y$ and L_2: $(2k + 1)y = 2x + 15$ do not intersect, where k is a constant. Find the value(s) of k.

3. Two straight lines L_1: $3x = 2(1 - my)$ and L_2: $6x - 8y + (n - 1) = 0$ have infinitely many points of intersection, where m and n are constants. Find the values of m and n. →Exercise 15.3: 19

4. Consider the following three straight lines:

L_1: $x + 2y = 6$

L_2: $y - 14 = \frac{4}{3}x$

L_3: $4x = 3y + 16$

Which of the following is true?

A. L_1 and L_2 do not intersect.

B. L_1 and L_3 intersect at a point.

C. L_2 and L_3 have infinitely many points of intersection.

D. L_1, L_2 and L_3 intersect at a point.

5. It is given that s is a constant. Three straight lines L_1: $y = -2$, L_2: $sy + s = 6x$ and L_3: $(s + 1)x - 4s = 5y$ cannot form a triangle. Find the values of s.

Learn at Home F4 Math worksheet 15.8

the Coordinates of the Point of Intersection of two straight lines

In this worksheet, give the answers for equations of straight lines in the general form.

(i) The coordinates of the point of intersection of two straight lines L_1 and L_2 can be found by solving the simultaneous equations $\begin{cases} \text{equation of } L_1 \\ \text{equation of } L_2 \end{cases}$.

(ii) The simultaneous equations can be solved by substitution or elimination.

1. The equations of two straight lines L_1 and L_2 are $2x - y + 6 = 0$ and $4x + 3y - 8 = 0$ respectively. L_1 and L_2 intersect at a point E.

(a) Find the coordinates of E.

(b) Find the equation of a straight line L_3 passing through E and with x-intercept 4.

→Exercise 15.3: 16

2. L_1: $3x - 5y - 9 = 0$, L_2: $x + 2y - 2 = 0$ and L_3: $x - 3y - 7 = 0$ are three straight lines. L_1 and L_3 intersect at K.

(a) Find the coordinates of K.

(b) A straight line L_4 passes through K and perpendicular to L_2. Find the equation of L_4.

→Exercise 15.3: 21

3. The straight lines L_1: $4x + 3y + 6 = 0$ and L_2: $x - ky + 7 = 0$ intersect at a point R, where k is a constant.

(a) Express the coordinates of R in terms of k.

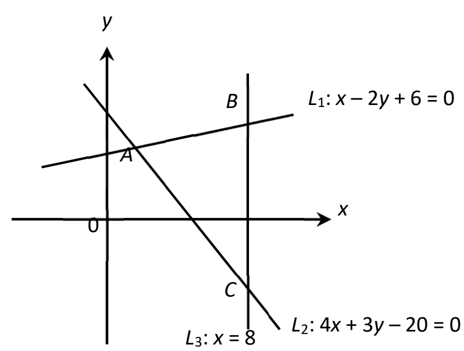
(b) If R is a point on the straight line L_3: $kx - y + 8 = 0$, find the value(s) of k.

→Exercise 15.3: 22

4. In the figure, L_1 and L_2 intersect at A. L_3 cuts L_1 and L_2 at B and C respectively. The equations of L_1, L_2 and L_3 are $x - 2y + 6 = 0$, $4x + 3y - 20 = 0$ and $x = 8$ respectively.

(a) Find the coordinates of A, B and C.

(b) Find the area of $\triangle ABC$.

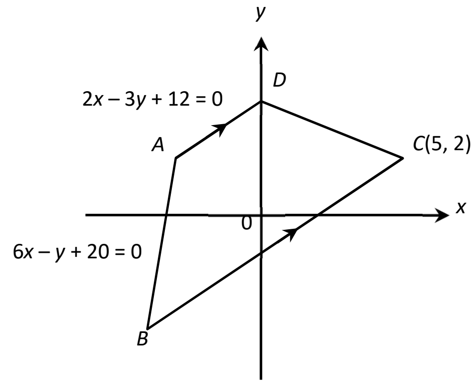


→Exercise 15.3: 24

5. In the figure, $ABCD$ is a trapezium with AD // BC. The equations of AB and AD are $6x - y + 20 = 0$ and $2x - 3y + 12 = 0$ respectively. The coordinates of C are (5, 2).

(a) Find the coordinates of B.

(b) Find the equation of diagonal BD.



Full Solution

https://qrgo.page.link/m6LvZ