Let's have a Math Chat

Equations of Straight Lines

- Equations of Straight Lines

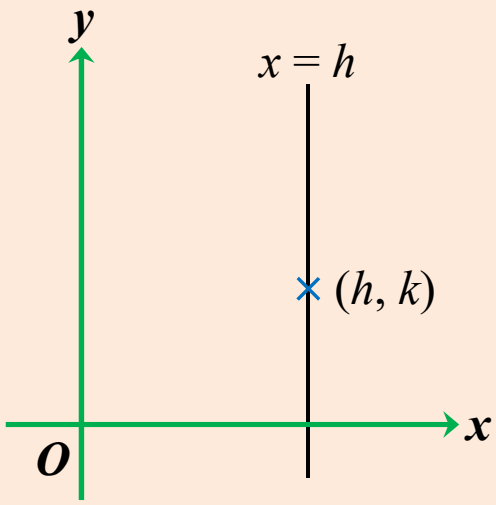
Equations of Straight Lines

Below are the ways of finding equations of different kinds of straight lines.

(1) Vertical Lines

The equation of a vertical line passing through point (h, k) is

$$x = h$$



Note:

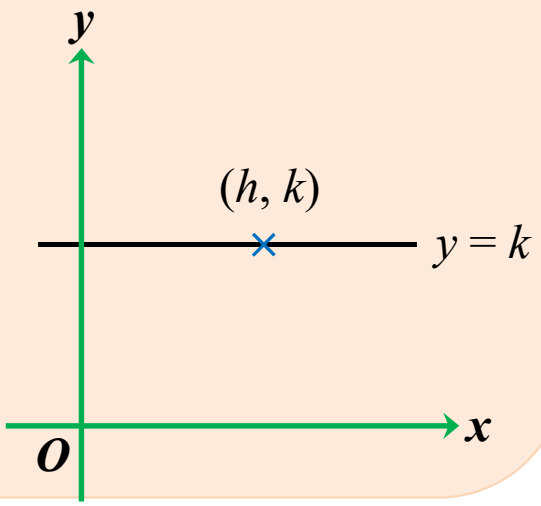
- Vertical lines are parallel to the y-axis.
- The equation of the y-axis is $x = 0$.

Equations of Straight Lines

(2) Horizontal Lines

The equation of a horizontal line passing through point (h, k) is

$$y = k$$



Note:

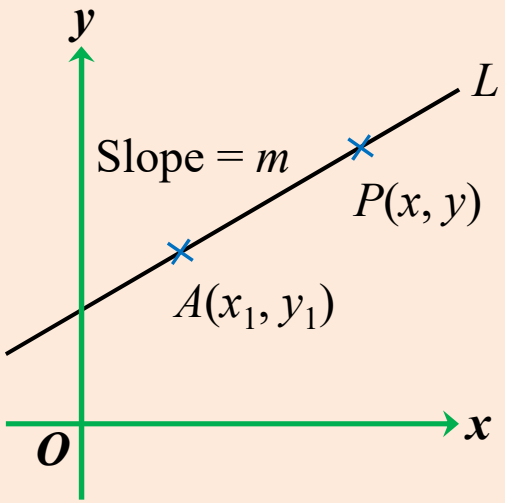
- Horizontal lines are parallel to the x-axis.
- The equation of the x-axis is $y = 0$.

Equations of Straight Lines

(3) Straight Lines

Given a point and the slope

Suppose the slope of a straight line L is m and L passes through a point $A(x_1, y_1)$.
If $P(x, y)$ is any point on L, then



$$\text{Slope of } AP = \frac{y - y_1}{x - x_1}$$

$$m = \frac{y - y_1}{x - x_1}$$

$$y - y_1 = m\,(x - x_1)$$

Therefore, the equation of a straight line passing through (x_1, y_1) with slope m is given by

$$y - y_1 = m\,(x - x_1)$$

This is called the **point-slope form** of the equation of a straight line.

Equations of Straight Lines

Example 1

Find the equation of the straight line passing through (3, –2) with slope –1.

Equation of the straight line:

$$\begin{aligned} y-(-2) &= -1\,(x-3) \\ y+2 &= -x+3 \\ y &= -x+1 \end{aligned}$$

Equations of Straight Lines

(3) Straight Lines

Given two points

If a straight line L passes through two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, we can find out the equation of L by the following steps:

(1) Find the slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

(2) Choose either P or Q and then apply the point-slope form.

Example 2

Find the equation of the straight line passing through $A(-3, 1)$ and $B(6, 10)$.

$$\text{Slope of the straight line} = \frac{10-1}{6-(-3)}$$

$$= \frac{9}{9}$$

$$= 1$$

Equation of the straight line:

$$y - 1 = 1\,[x - (-3)]$$

$$y - 1 = x + 3$$

$$y = x + 4$$

Equations of Straight Lines

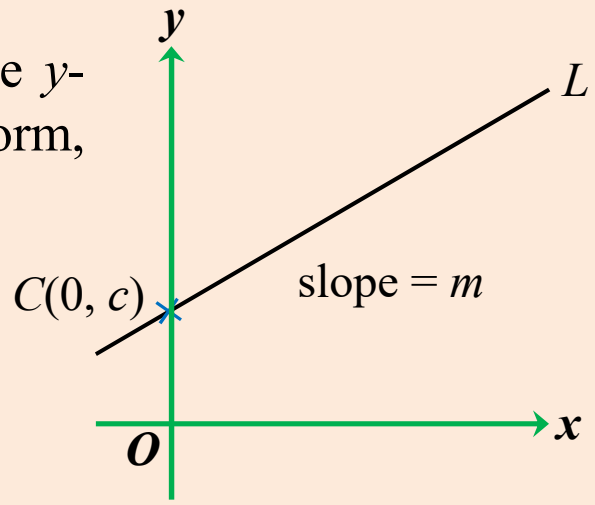
(3) Straight Lines

Given the slope and the y-intercept

If the slope of a straight line L is m and the y-intercept is c, then by using the point-slope form, we have

$$y - c = m(x - 0)$$

$$y = mx + c$$



Therefore, the equation of a straight line with slope m and y-intercept c is given by

$$y = mx + c$$

This is called the **slope-intercept form** of the equation of a straight line.

Example 3

Find the equation of the straight line with y-intercept 5 and slope 3.

Equation of the straight line:

$$y = 3x + 5$$

Equations of Straight Lines

(3) Straight Lines

Given the x-intercept and the y-intercept

If the x-intercept and the y-intercept of a straight line L are a and b respectively, i.e. L passes through $(a, 0)$ and $(0, b)$. We can find the equation of L by the following steps:

(1) Find the slope $m = \dfrac{b-0}{0-a}$.

(2) Apply the slope-intercept form.

Example 4

Find the equation of the straight line with x-intercept 2 and y-intercept –3.

$$\text{Slope of the straight line} = \frac{-3-0}{0-2} = \frac{-3}{-2} = \frac{3}{2}$$

Equation of the straight line:

$$y = \frac{3}{2}x - 3$$