Let's have a Math Chat

Intersection of Two Straight Lines

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We have three possible cases concerning the intersection of two straight lines.

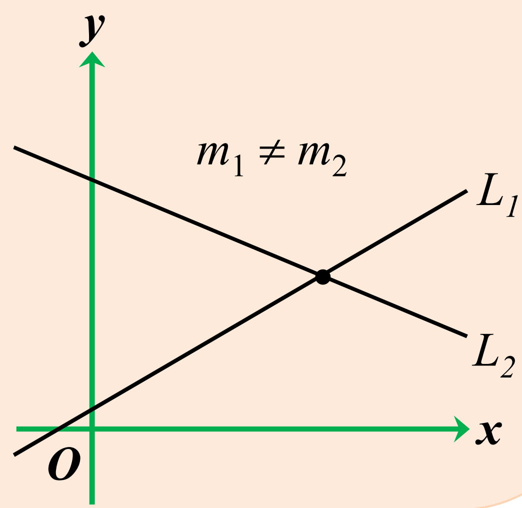
Consider two straight lines L_1 and L_2 lying on the same rectangular coordinate plane.

Let

m_1 and c_1 be the slope and the y-intercept of L_1 respectively,
m_2 and c_2 be the slope and the y-intercept of L_2 respectively.

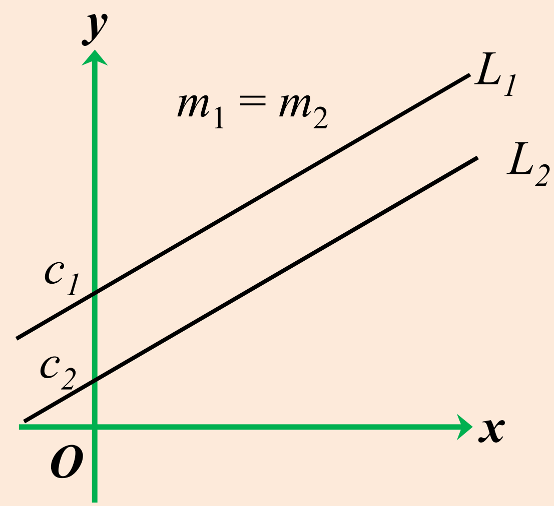
(1) There is only one point of intersection.

L_1 and L_2 have different slopes,
i.e. $m_1 \neq m_2$.



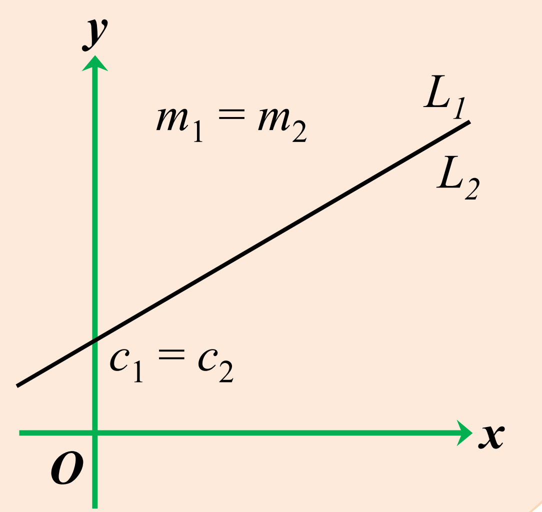
(2) There are no points of intersection.

L_1 and L_2 have the same slope but different y-intercept,
i.e. $m_1 = m_2$ and $c_1 \neq c_2$.
In this case, L_1 and L_2 are parallel to each other.



(3) There are infinitely many points of intersection.

L_1 and L_2 have the same slope and the same y-intercept,
i.e. $m_1 = m_2$ and $c_1 = c_2$.
In this case, L_1 and L_2 overlap with each other. The equation of L_1 and L_2 represent the same line.



Intersection of Two Straight Lines

Example 1

In each of the following, find the number of points of intersection of L_1 and L_2. When L_1 and L_2 intersect at only one point, find the coordinates of the point of intersection.

(a) $L_1: y = -3x + 7$ $\quad$ $L_2: 3x + y + 7 = 0$

(b) $L_1: x - 4y + 3 = 0$ $\quad$ $L_2: 8y = 2x + 6$

(c) $L_1: 5x + 2y - 8 = 0$ $\quad$ $L_2: 2y = 5x + 8$

(a) Slope of $L_1 = -3$ $\quad$ Slope of $L_2 = -\frac{3}{1} = -3$

y-intercept of $L_1 = 7$ $\quad$ y-intercept of $L_2 = -\frac{7}{1} = -7$

L_1 and L_2 have the same slope but different y-intercepts.

$\therefore$ There are no points of intersection.

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(c) $L_1: 5x + 2y - 8 = 0$ $\quad$ $L_2: 2y = 5x + 8$

(b) Slope of $L_1 = -\frac{1}{-4} = \frac{1}{4}$

y-intercept of $L_1 = -\frac{3}{-4} = \frac{3}{4}$

Slope of $L_2 = \frac{2}{8} = \frac{1}{4}$

y-intercept of $L_2 = \frac{6}{8} = \frac{3}{4}$

L_1 and L_2 have the same slope and the same y-intercept.

$\therefore$ There are infinitely many points of intersection.

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(c) $L_1: 5x + 2y - 8 = 0$ $L_2: 2y = 5x + 8$

(c) Slope of $L_1 = -\frac{5}{2}$ Slope of $L_2 = \frac{5}{2}$

L_1 and L_2 have different slopes.

$\therefore$ There is only one point of intersection.

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Example 1

In each of the following, find the number of points of intersection of L_1 and L_2. When L_1 and L_2 intersect at only one point, find the coordinates of the point of intersection.

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(b) $L_1: x - 4y + 3 = 0$ $L_2: 8y = 2x + 6$

(c) $L_1: 5x + 2y - 8 = 0$ $L_2: 2y = 5x + 8$

(c) L_1: $5x + 2y - 8 = 0$ .... (1)

L_2: $2y = 5x + 8$ .... (2)

Put (2) into (1). $5x + (5x + 8) - 8 = 0$

$$10x = 0$$

$$x = 0$$

Put $x = 0$ into (2). $2y = 5(0) + 8$

$$y = 4$$

∴ The coordinates of the point of intersection are (0, 4).

Example 2

Consider two straight lines L_1 and L_2,

L_1: $5x + 2y - 14 = 0$ .... (1)

L_2: $x - 3y - 13 = 0$ .... (2)

It is given that L_1 and L_2 intersect at a point P.

(a) Find the coordinates of P.

(b) Find the equation of a straight line L_3 passing through P and perpendicular to the line L_4: $4x - y + 6 = 0$.

(a) (2) × 5: $5x - 15y - 65 = 0$ .... (3)

(1) – (3): $(5x + 2y - 14) - (5x - 15y - 65) = 0$

$17y + 51 = 0$

$y = -3$

Put $y = -3$ into (2). $x - 3(-3) - 13 = 0$

$x = 4$

∴ The coordinates of P are (4, –3).

Example 2

Consider two straight lines L_1 and L_2,

L_1: $5x + 2y - 14 = 0$ .... (1)

L_2: $x - 3y - 13 = 0$ .... (2)

It is given that L_1 and L_2 intersect at a point P.

(a) Find the coordinates of P.

(b) Find the equation of a straight line L_3 passing through P and perpendicular to the line L_4: $4x - y + 6 = 0$.

(b) Slope of $L_4 = -\frac{4}{-1} = 4$

$\therefore$ Slope of $L_3 = -\frac{1}{4}$

Equation of L_3:

$$y - (-3) = -\frac{1}{4}(x - 4)$$

$$4y + 12 = -x + 4$$

$$x + 4y + 8 = 0$$