Learn at Home with Diverse Online Resources

In view of the impact brought by the new coronavirus, schools in Hong Kong have delayed school resumption after the Lunar New Year Holiday. Let's make good use of this online resources to facilitate us to learn at home.

Class: F.5S

F5 Mathematics Chapter 21 Measures of Dispersion

12 February, 2020

Before finish the following excises, let's have a Math chat by scanning the QR code

21.1	Range and Inter-quartile Range	https://qrgo.page.link/jNHdg
21.2	Box-and-Whisker Diagram	https://qrgo.page.link/ex1Zr
21.3	Standard Deviation	https://qrgo.page.link/iptJW
21.4	Applications of Standard Deviation	https://qrgo.page.link/GKcR8
21.5	Effects on Dispersion with Changes in Data	https://qrgo.page.link/b5wxh

Exercises 1 on Measures of Dispersion (Ch.21.1 Range and Inter-quartile Range)

1. Find the range of each of the following sets of data.

(a) -4, 4, -7, 9, -2, 3	(b) 58, 82, 78, 61, 33, 49, 47
(c) -0.8, -1.0, 1.3, 0.2, -0.9, -2.9, -1.7, -2.1	

2. The table shows the expenses of 20 people on a day.

3.

Expense (\$) 30 – 39		40 - 49	50 - 59	60 - 69	
Frequency 3		9	6	2	

Find the inter-quartile range of each of the following sets of data.

Find the range of the expenses of the people on that day.

(a)	5, 7, 10, 13, 15, 16	(b)	2, 7, 9, 12, 13, 18, 23
(c)	20, 23, 27, 28, 36, 38, 42, 51	(d)	15, 20, 23, 28, 32, 36, 41, 46, 48

4. Find the inter-quartile range of each of the following sets of data.

(c) 25.3, 22, 26.2, 22.7, 21.5, 14.7, 18.9, 14.1

(d) x-3, x+1, x+2, x-1, x-5, x, x-1, x+4, x-2

5. The waiting time (in min) of 9 patients in a clinic are shown below. 17, 22, 24, 18, 22, 28, 23, 29, 24

- (a) Find the range of the waiting time of the patients.
- (b) Find the inter-quartile range of the waiting time of the patients.

- 6. The daily attendance of two tutorial classes in a week are shown below.
 - Class *A*: 49, 41, 39, 46, 36, 42, 42 Class *B*: 39, 38, 45, 36, 42, 35, 36
 - (a) Find the range of the daily attendance of each class in the week.

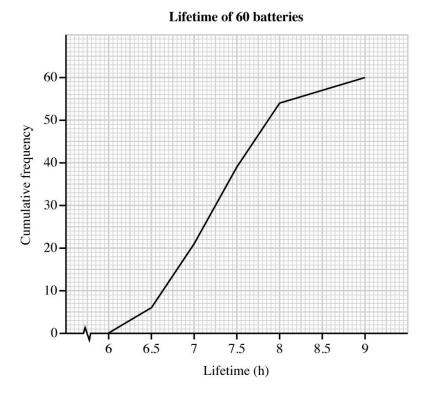
Class A:

Class B:

- (b) By comparing the ranges found in (a), which class has a smaller dispersion in the daily attendance? Explain your answer.
- Consider the following two sets of data.
 Set A: 75, 55, 46, 79, 84, 58, 63, 72
 Set B: 87, 75, 58, 77, 53, 84, x, 58
 - (a) Find the values of x if the two sets of data have the same range.

(b) For each of the values of x found in (a), find the inter-quartile range of set B.

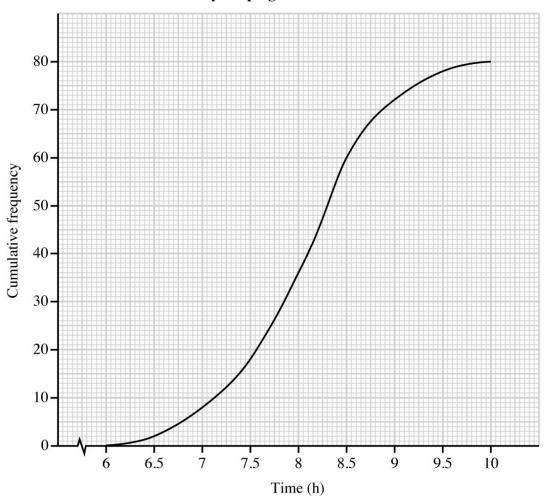
8. The figure shows the cumulative frequency polygon of the lifetime of 60 batteries.



(a) Find the median, the first quartile and the third quartile of the lifetime of the batteries.

(b) Find the inter-quartile range of the lifetime of the batteries.

9. The figure shows the cumulative frequency curve of the daily sleeping hours of 80 students.



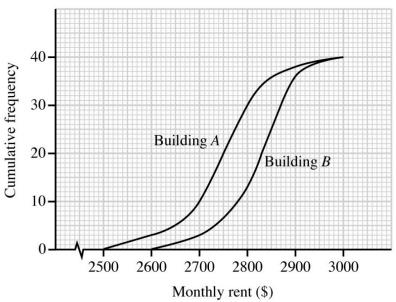
Daily sleeping hours of 80 students

(a) Find the median, the first quartile and the third quartile of the daily sleeping hours of the students.

Median = $Q_1 = Q_3 =$

(b) Find the inter-quartile range of the daily sleeping hours of the students.

10. The figure shows the cumulative frequency curves of the monthly rents of the flats in buildings A and B in a year.



Monthly rents of the flats in two buildings A and B in a year

(a) Find the median, the first quartile and the third quartile of monthly rents of the flats of each building.

Building A	Building B
Median =	Median =
$Q_1 =$	$Q_1 =$
$Q_3 =$	$Q_3 =$

(b) (i) Find the inter-quartile range of the monthly rents of the flats of each building.

(ii) By comparing the inter-quartile ranges found in (b)(i), which building has a smaller dispersion in the monthly rents of the flats?

11. The stem-and-leaf diagram shows the maximum daily temperatures (in °C) in June.

	Leaf (0.1°C)							
28	4	8	9					
29	1	1	1	4	6	7	8	9
30	2	3	4	x	7			
31	0	1	5	6	7	8	у	
32	3	5						
28 29 30 31 32 33	0	4	5	8	8			

It is known that the median and the inter-quartile range of the maximum daily temperatures in June are 30.6° C and 2.3° C respectively. Find the values of x and y.

12. The stem-and-leaf diagram shows the scores of 30 students of class 5A in an examination.

Stem (10 marks)								
4	0	4	7					
5	1	2	3	3	6	9		
6	0	1	1	2	5	7	8	9
7	1	2	2	3	4	6	7	9
8	0	3	8					
4 5 6 7 8 9	4	5						

- (a) Find the range of the scores of the students of class 5A.
- (b) Find the inter-quartile range of the scores of the students of class 5A.
- (c) A student is selected randomly from class 5A. Find the probability that the student selected scores
 - (i) 70 or above, (ii) 60 or below.

13. The stem-and-leaf diagram shows the hourly wages of the employees of two companies *A* and *B*.

Company A	<u>Company</u>				
Leaf (\$1)	Stem (\$10)	Leaf (\$1)			
5 9 7 3 2 2 2	2 3 4 5 6	9 4 x 0 7 y 5 0			

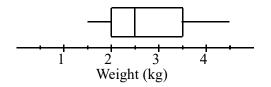
It is known that the mean hourly wages of the employees of both companies are the same and the inter-quartile range of the hourly wages of the employees of company B is \$15.

(a) Find the values of x and y.

- (b) Find the inter-quartile range of the hourly wages of the employees from company A.
- (c) By comparing the inter-quartile ranges found in (a), which company has a larger dispersion in the hourly wages of the employees?

Exercises 2 on Measures of Dispersion (Ch.21.2 Box-and-whisker Diagrams)

1. The box-and-whisker diagram shows the weights of newborn babies in a hospital on a day. Find the median, the range and the inter-quartile range of their weights.



Median =

Range =

Inter-quartile range =

2. The stem-and-leaf diagram shows the time spent daily by John on reading books in 14 days.

Stem (10 min)	Leaf (1 min)						
2 3 4	5	5	8	9			
3	0	1	1	3	4	5	7
4	1	1	2				

(a) Find the minimum, the maximum, the median, the first quartile and the third quartile of the time spent daily by John on reading books.

Minimum =	$Q_1 =$
Maximum =	$Q_3 =$
Median =	

(b) Draw a box-and-whisker diagram to describe the time spent daily by John on reading books.

 $24 \quad 26 \quad 28 \quad 30 \quad 32 \quad 34 \quad 36 \quad 38 \quad 40 \quad 42$

3. The figure shows the cumulative frequency curve of the monthly salaries of 80 people.



(a) Find the minimum, the maximum, the median, the first quartile and the third quartile of the monthly salaries of the people.

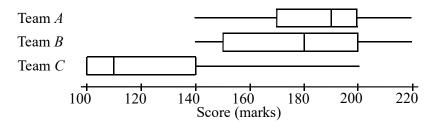
Minimum = $Q_1 =$ Maximum = $Q_3 =$ Median =

(b) Draw a box-and-whisker diagram to describe the monthly salaries of the people.

6	8	10	12	14	16	18	20	22	24
		6					-		

(Thousand dollars)

4. The box-and-whisker diagrams show the scores obtained by 3 bowling teams in a competition.



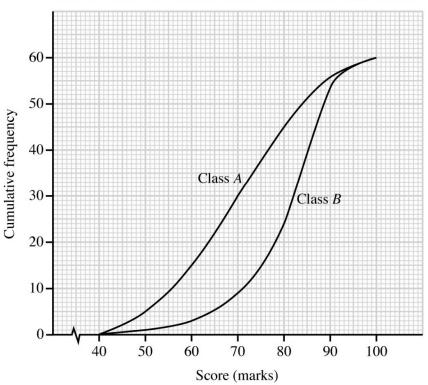
(a) Which team has the largest range of scores?

(b) Which team has the smallest inter-quartile range of scores?

(c) Which team has the highest median score?

(d) Which team performs the best? Explain your answer briefly.

5. The figure shows the cumulative frequency curves of the scores of students in classes *A* and *B* in an examination.



Scores of students in two classes A and B in an examination

(a) Draw box-and-whisker diagrams in the same figure to describe the scores of the students of Classes A and B respectively.

Class A

Class B

 $40 \ 45 \ 50 \ 55 \ 60 \ 65 \ 70 \ 75 \ 80 \ 85 \ 90 \ 95 \ 100$

(b) Which class has a larger dispersion in scores in the examination? Explain your answer briefly.

(c) Which class does better in the examination? Explain your answer briefly.

Exercises 3 on Measures of Dispersion (Ch.21.3 Standard Deviation)

In this exercise, give the answers correct to 3 significant figures if necessary.

- 1. The mean of six numbers 4, 8, 7, *k*, 12, 9 is 9.
 - (a) Find the value of k.

- (b) Find the standard deviation of the six numbers.
- 2. The table shows the numbers of sick leaves taken by the 30 employees of a company in a year.

Number of sick leaves taken	0	1	2	3	4	5	
Frequency	5	8	4	9	3	1	

Find the mean and the standard deviation of the numbers of sick leaves taken.

3. The table shows the numbers of movies watched by 40 people in a month.

Number of movies watched	0	1	2	3	4	5
Frequency	3	6	10	13	x	3

Find the mean and the standard deviation of the numbers of movies watched by the people in that month.

4. The table shows the areas of 50 flats.

Area (m ²)	Class mark (m ²)	Frequency
40-44		13
45 – 49		14
50 - 54		11
55 - 59		6
60 - 64		4
65 - 69		2

- (a) Complete the above table.
- (b) Find the mean and the standard deviation of the areas of the flats.

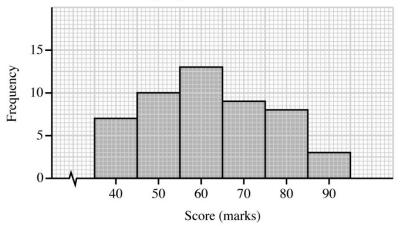
5.	The table shows	the daily	profits of a	restaurant in April	l.
----	-----------------	-----------	--------------	---------------------	----

Daily profit (thousand dollars)	Class mark (thousand dollars)	Frequency
4.5 – 4.9		5
5.0-5.4		10
5.5 - 5.9		
6.0-6.4		4
6.5 - 6.9		5

(a) Complete the above table.

(b) Find the mean and the standard deviation of the daily profits of the restaurant in April.

6. The histogram shows the scores of a group of students in an examination.



Scores of a group of students in an examination

- (a) Find the total number of students in the group.
- (b) Find the mean and the standard deviation of the scores of the students of the group.

	Day 1	Day 2	Day 3	Day 4	Day 5
Car A	7.4 L	6.4 L	7.3 L	7.9 L	7.2 L
Car B	7.8 L	8.1 L	7.1 L	6.2 L	6.8 L

7. The table shows the amounts of petrol consumed by two cars *A* and *B* in five days.

- (a) Find the mean and the standard deviation of the amounts of petrol consumed by each car.
- (b) Using the results of (a), which car has a larger dispersion in the amounts of petrol consumed.

8. The table shows the numbers of students wearing glasses in 30 classes in school A.

Number of students	0-4	5 – 9	10 - 14	15 – 19	20 - 24
Frequency	2	12	10	x	1

(a) Find the value of x.

- (b) Find the mean and the standard deviation of the numbers of students wearing glasses in 30 classes.
- (c) The standard deviation of the numbers of students wearing glasses in 30 classes in school *B* is 3. Which school has a larger dispersion in the numbers of students wearing glasses?

Exercises 4 on Measures of Dispersion (Ch.21.4 Applications of Standard Deviation)

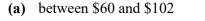
1. The mean and the standard deviation of the time spent daily on internet by a group of students are 2 h and 0.5 h respectively. Find the standard score of a student of the group who spent 3 h daily on internet.

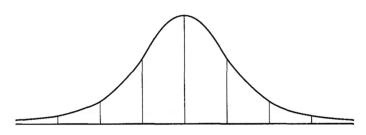
2. The mean and the standard deviation of the diameters of a batch of disks are 20 cm and 0.2 cm respectively. Find the standard score of a disk of diameter 19.95 cm.

3. The standard deviation of the weights of puppies in a pet shop was 0.4 kg. The weight of a puppy in the shop was 2.8 kg and its standard score was 1.5. Find the mean weight of the puppies.

4. The mean price of the dolls in a shop was \$250. The price of a doll was \$150 and its standard score was -1.25. Find the standard deviation of the prices of the dolls.

- 5. In last year, the daily maximum temperatures of a city were normally distributed with mean 26°C and standard deviation 4°C. Find the percentage of days in last year on which the daily maximum temperatures were
 - (a) between 18°C and 34°C 34% 13.5% (b) between 26°C and 34°C 2.35% (c) below 18°C (d) between 22°C and 34°C
- On Valentine's Day, the prices of the presents received by a group of girls are normally distributed 6. with mean \$60 and standard deviation \$14. Find the percentage of girls in the group who receive a present of price





(e) between 18° C and 22° C

- **(b)** less than \$18
- (c) between \$74 and \$102

(d) between \$18 and \$32

(e) between \$18 and \$88

- 7. The weights of garbage dumped by 2400 families on a day are normally distributed with mean 7.6 kg and standard deviation 1.3 kg. Find the number of families whose weights of garbage dumped on that day are
 - (a) between 6.3 kg and 10.2 kg (b) between 8.9 kg and 10.2 kg

(c) less than 6.3 kg or more than 8.9 kg

(d) less than 5 kg or more than 8.9 kg

- 8. The amounts of money spent by the visitors in a theme park on a day were normally distributed with mean \$840 and standard deviation \$120. There were 400 visitors who spent more than \$1080 on that day.
 - (a) Find the total number of visitors in the theme park on that day.

(b) Find the number of visitors who spent less than \$720.

- **9.** The amounts of water consumed on a day by a group of people were normally distributed with mean 1680 mL and standard deviation 280 mL. It was known that 60 people consumed less than 1120 mL of water on that day.
 - (a) Find the total number of people in the group.

(b) Find the number of people in the group whose amounts of water consumed on that day were less than 1400 mL or more than 1960 mL.

10. The mean age of the patients of a clinic was 35. May was a patient in that clinic. She was 47 years old and her standard score was 0.8.

(a) Find the standard deviation of the ages of the patients of the clinic.

(b) Joanna is another patient of the clinic and her standard score was -1.2. Find her age.

- 11. The numbers of passengers picked by 2000 taxis on a day are normally distributed with standard deviation 5. Taxi *A* picked 22 passengers on that day and its standard score was 0.6.
 - (a) Find the mean number of passengers picked by the taxis on that day.

(b) If taxi *B* picked 15 passengers on that day, find its standard score.

12. In an examination, the mean and the standard deviation of the scores in Chemistry and Biology of a class were given below.

Subject	Mean	Standard deviation
Chemistry	56 marks	7 marks
Biology	52 marks	5 marks

Janet got 70 marks in Chemistry and 60 marks in Biology.

(a) Find the standard score of Janet in each subject.

(b) In which subject did Janet perform better by comparing the standard scores?

13. In an examination, the mean and the standard deviation of the scores in Chinese and English of a class were given below.

Subject	Mean	Standard deviation
Chinese	65 marks	15 marks
English	47 marks	6 marks

(a) Dennis got 86 marks in Chinese and 59 marks in English. In which subject did he perform better by comparing the standard scores?

(b) Relative to the performance of the class, what marks in Chinese should be equivalent to 59 marks in English?

- **14.** In an examination, Joyce and Phoebe got 51 marks and 81 marks respectively. The standard scores of Joyce and Phoebe are -1.2 and 0.8 respectively.
 - (a) Find the mean and the standard deviation of the scores in the examination.

(b) Kelvin got 90 marks in the examination. Find his standard score.

15. In a batch of bottles of milk, the volumes of milk in the bottles are normally distributed. There are 2.5% of bottles with volumes of milk more than 316 mL and 0.15% of bottles with volumes of milk less than 274 mL. Find the mean and the standard deviation of the volumes of milk in the bottles.

- **16.** In a test, the scores of the candidates are normally distributed with mean 61 marks and standard deviation 9 marks. The top 2.5% of the candidates get grade 'A' in the test.
 - (a) Find the minimum standard score of the candidates who get grade 'A' in the test.

(b) Find the minimum score of the candidates who get grade 'A' in the test.

- 17. The weights of a group of adults are normally distributed. Sandy and Roger are two people in the group with weights 40 kg and 86 kg respectively. The standard scores of Sandy and Roger are -0.8 and 1.5 respectively. It is known that 100 adults in the group weigh more than 96 kg.
 - (a) Find the mean and the standard deviation of the weights of the adults.
 - (b) Find the number of adults in the group.
 - (c) The adults who weigh less than 36 kg or more than 76 kg are advised to take a body check. Find the number of adults who are advised to take a body check.

Exercises 5 on Measures of Dispersion (Ch.21.5 Effects on Dispersion with Changes in Data)

In this exercise, give the answers correct to 3 significant figures if necessary.

1. In a singing contest, the scores of 15 contestants in the first round are given below.

15, 17, 13, 17, 8, 6, 14, 18, 17, 13, 12, 10, 19, 9, 7

- (a) Find the range, the inter-quartile range and the standard deviation of the scores of the contestants.
- (b) The contestants got 10 marks or above can proceed to the second round of the contest. Find the range, the inter-quartile range and the standard deviation of the scores of the contestants proceed to the second round.

- 2. The ages of the employees of a company are 20, 25, 28, 31, 38, 47 and 50 respectively.
 - (a) Find the range and the inter-quartile range of the ages of the employees.

(b) An employee resigns from the company. The range and the inter-quartile range of the ages of the remaining employees are the same as the results of (a). Find the possible age(s) of the resigned employee.

- **3.** The marked prices of the diamond rings in a jewellery shop are \$8000, \$9000, \$17 500, \$30 000, \$23 000, \$15 500, \$24 600 and \$12 400 respectively.
 - (a) Find the mean, the range, the inter-quartile range and the standard deviation of the marked prices of the diamond rings.

(b) A diamond ring is sold such that the mean of the marked prices of the remaining diamond rings is the same as the mean found in (a). How do the range, the inter-quartile range and the standard deviation change when compared with the results of (a)?

- 4. In a 9-day trip to Shanghai, the numbers of photos taken by Tony on the first 8 days are 65, 67, 68, 49, 57, 61, 54 and 59 respectively.
 - (a) Find the range and the inter-quartile range of the numbers of photos on the first 8 days of the trip.

(b) Tony also took photos on the last day of the trip. The range and the inter-quartile range of the numbers of photos taken in the whole trip are the same as the results of (a). Find the possible number(s) of photos taken on the last day.

- 5. The monthly expenses of Jacky from January to May are given below. \$4200, \$5000, \$3800, \$6400, \$3600
 - (a) Find the range, the inter-quartile range and the standard deviation of the monthly expenses of Jacky from January to May.

(b) The expense of Jacky in June in that year is \$5800. Find the range, the inter-quartile range and the standard deviation of the monthly expenses of Jacky from January to June.

6. The heights of the students of a class are shown in the stem-and-leaf diagram below.

Stem (10 cm)								
14 15 16 17	0	1	4	5	6			
15	0	1	2	5	7	9		
16	0	2	3	5	6	7	9	
17	2	6						

(a) Find the range, the inter-quartile range and the standard deviation of the heights of the students.

(b) A student of height 199 cm is added to the class. How do the range, the inter-quartile range and the standard deviation change when compared with the results of (a)?

- 7. A travel agency organizes a 2-day trip to Macau in each month. The numbers of people joined the trips in the last 12 months are 14, 14, 17, 18, 12, 11, 16, 18, 13, 16, 12 and 19 respectively.
 - (a) Find the range, the inter-quartile range and the standard deviation of the numbers of people joined the trips in the last 12 months.

(b) The cost C (in thousand dollars) in organizing the trip is given by C = 2X + 9, where X is the number of people joined the trip. Find the range, the inter-quartile range and the standard deviation of the costs in organizing the trips in the last 12 months.

- **8.** Philip took part in a game. The mean and the standard deviation of the scores got by the participants of the game are 27 marks and 5 marks respectively. The standard score of Philip is 0.3.
 - (a) Find the score of Philip.

(b) Will the standard score of Philip change if the scores of all the participants are doubled? Explain your answer briefly.

- **9.** The mean, the range and the standard deviation of the weights of a group of adults are 66 kg, 34 kg and 10 kg respectively. Peter is a member of the group and his weight is 78 kg.
 - (a) Find the standard score of Peter.

- (b) A person of weight 66 kg left the group.
 - (i) How do the mean, the range and the standard deviation of the weights of the members of the group change after the person left?

(ii) How do the standard score of Peter change after the person left? Explain your answer briefly.

- **10.** A researcher conducted a study on the numbers of books borrowed from the school library last year by 14 students. The results are 19, 11, 7, 8, 13, 3, 5, 4, 19, 9, 14, 8, 3 and 14.
 - (a) Find the median and the inter-quartile range of these data.
 - (b) The numbers of books borrowed from the school library by another 3 students is added to the study of the researcher. Each of them borrows more than 20 books.
 - (i) Find the new median and the new inter-quartile range.
 - (ii) Describe briefly the change in the median and the inter-quartile range of the numbers of books borrowed from the school library.
 - (iii) A datum *y* is discarded from the set of data because it is wrongly recorded. The inter-quartile range of the remaining data is the same as that of the set of data before discarding the datum *y*. Find the possible value(s) of *y*.

https://qrgo.page.link/7Ao4q

