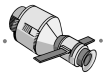
Chapter 7 Indefinite Integrals (I)

7.1 Concept of Indefinite Integrals and Integration Formulae of Algebraic Functions



Quick Check

Fill in the blanks. **(1 – 6)**

1. $\int 5x\,dx = \_\_\_\_ \int x\,dx$

2. $\int (x + x^2)\,dx = \int x\,dx \_\_\_\_ \int x^2\,dx$

3. $\int (3x^2 - x)\,dx = \_\_\_\_ \int x^2\,dx \_\_\_\_ \int x\,dx$

4. $\int 7\,dx = \_\_\_\_ + C$, where C is an arbitrary constant.

5. $\int x^2\,dx = \dfrac{x^{\square}}{\square} + \_\_\_\_\_$

6. $\int \dfrac{1}{x}dx = \_\_\_\_\_$

Level 1

Find the following indefinite integrals. **(1 – 12)**

1. $\int -4\,dx$ **2.** $\int \dfrac{2}{3}dx$ **3.** $\int x\,dx$

4. $\int x^3\,dx$ **5.** $\int 2x^{-7}\,dx$ **6.** $\int \dfrac{4}{x^9}dx$

7. $\int 8\sqrt[4]{x}\,dx$ **8.** $\int (1 - 2x^3)\,dx$ **9.** $\int (8x - x^2)\,dx$

10. $\int (x^4 + 3x^2)\,dx$ **11.** $\int \dfrac{5}{x}dx$ **12.** $\int \left(4 - \dfrac{1}{x}\right)dx$

Find the following indefinite integrals. **(13 – 30)**

13. $\int (3x^2 - 2)\,dx$ **14.** $\int (6x^5 - 8x^3 + 4x)\,dx$ **15.** $\int (4x^2 - 5x + 1)\,dx$

16. $\int \left(9 - \dfrac{5}{x}\right)dx$ **17.** $\int \left(x^3 - \dfrac{2}{x}\right)dx$ **18.** $\int \left(6x^2 - x + \dfrac{1}{x}\right)dx$

19. $\int \left(x^3 + 3x^2 + \dfrac{1}{x}\right)dx$ **20.** $\int x(x + 8)\,dx$ **21.** $\int x(4x + 7)\,dx$

22. $\int 2x(x^2 - 3)\,dx$ **23.** $\int [x^3(x^2 - 5x) + x^4]\,dx$ **24.** $\int [x(x^2 - 2) - x + 3]\,dx$

25. $\int x^2\left(4x-\frac{2}{x}\right)dx$ **26.** $\int 5x^2\sqrt{x}\,dx$ **27.** $\int x^2(\sqrt[4]{x})\,dx$

28. $\int \frac{5+3x}{x}dx$ **29.** $\int \frac{7x^2-4x}{2x}dx$ **30.** $\int \frac{x^3-4x^2+4x}{x}dx$

Level 2

Find the following indefinite integrals. **(31 – 52)**

31. $\int\left(2x+\frac{9}{\sqrt{x}}\right)dx$ **32.** $\int\left(\frac{2}{\sqrt[3]{x}}-x^2\right)dx$

33. $\int\left(5\sqrt{x}-\frac{1}{\sqrt[4]{x}}\right)dx$ **34.** $\int(1+6x)^2\,dx$

35. $\int(5x-3)^2\,dx$ **36.** $\int(2x-7)(2x+7)\,dx$

37. $\int(x+2)(5x-2)\,dx$ **38.** $\int(3x+4)(6x+1)\,dx$

39. $\int\left(\frac{x}{4}+\frac{4}{x}\right)^2dx$ **40.** $\int\left(2x-\frac{1}{x^2}\right)^2dx$

41. $\int\frac{\sqrt{x}}{x^4}dx$ **42.** $\int\sqrt{x}(6x-1)\,dx$

43. $\int(\sqrt{x}-3)(\sqrt{x}-4)\,dx$ **44.** $\int(x^{-\frac{1}{3}}+2)^2\,dx$

45. $\int\frac{2x^4-x^2-3}{x^2}dx$ **46.** $\int\frac{x^5-3x^4+4x}{x^6}dx$

47. $\int\frac{7x+11}{x}dx$ **48.** $\int\frac{x^4-1}{x}dx$

49. $\int\frac{3-x}{x^2}dx$ **50.** $\int\frac{x^3-5}{x^4}dx$

51. $\int\frac{(x-5)(6x-1)}{x}dx$ **52.** $\int\frac{(5-x)(3x-2)}{x^3}dx$

53. Find $\int(2x+a)(3x-b)\,dx$, where a and b are constants.

54. **(a)** Consider $y = \sqrt{\sqrt{2x} - 3}$. Find $\dfrac{dy}{dx}$.

(b) Using the result of **(a)**, find $\displaystyle\int \frac{5}{2(\sqrt{\sqrt{2x} - 3})(\sqrt{2x})}\,dx$.

55. **(a)** Consider $y = \dfrac{x^2 + 2}{(x^2 + 4)^2}$. Find $\dfrac{dy}{dx}$.

(b) Using the result of **(a)**, find $\displaystyle\int \left(\frac{x}{x^2 + 4}\right)^3 dx$.

56. Let m be a positive integer.

(a) Show that $\dfrac{d}{dx}[x^m(m \ln x - 1)] = m^2 x^{m-1} \ln x$.

(b) Hence, show that $\displaystyle\int x^{m-1} \ln x\,dx = \frac{1}{m}x^m \ln x - \frac{x^m}{m^2} + C$, where C is an arbitrary constant.

(c) Using the result of **(b)**, find $\displaystyle\int x^3 \ln 2x\,dx$.

Chapter 7 Answer Key

Section 7.1

Quick Check

1. $5\int x\,dx$

2. $\int x\,dx + \int x^2\,dx$

3. $3\int x^2\,dx - \int x\,dx$

4. $7x + C$, where C is a constant

5. $\frac{x^3}{3} + C$, where C is a constant

6. $\ln|x| + C$, where C is a constant

Level 1

1. $-4x + C$

2. $\frac{2x}{3} + C$

3. $\frac{x^2}{2} + C$

4. $\frac{x^4}{4} + C$

5. $-\frac{1}{3}x^{-6} + C$

6. $-\frac{1}{2}x^{-8} + C$

7. $\frac{32}{5}x^{\frac{5}{4}} + C$

8. $-\frac{x^4}{2} + x + C$

9. $-\frac{x^3}{3} + 4x^2 + C$

10. $\frac{x^5}{5} + x^3 + C$

11. $5\ln|x| + C$

12. $4x - \ln|x| + C$

13. $x^3 - 2x + C$

14. $x^6 - 2x^4 + 2x^2 + C$

15. $\frac{4x^3}{3} - \frac{5x^2}{2} + x + C$

16. $9x - 5\ln|x| + C$

17. $\frac{x^4}{4} - 2\ln|x| + C$

18. $2x^3 - \frac{x^2}{2} + \ln|x| + C$

19. $\frac{x^4}{4} + x^3 + \ln|x| + C$

20. $\frac{x^3}{3} + 4x^2 + C$

21. $\frac{4x^3}{3} + \frac{7x^2}{2} + C$

22. $\frac{x^4}{2} - 3x^2 + C$

23. $\frac{x^6}{6} - \frac{4x^5}{5} + C$

24. $\frac{x^4}{4} - \frac{3x^2}{2} + 3x + C$

25. $x^4 - x^2 + C$

26. $\frac{10}{7}x^{\frac{7}{2}} + C$

27. $\frac{4}{13}x^{\frac{13}{4}} + C$

28. $5\ln|x| + 3x + C$

29. $\frac{7x^2}{4} - 2x + C$

30. $\frac{x^3}{3} - 2x^2 + 4x + C$

Level 2

31. $x^2 + 18x^{\frac{1}{2}} + C$ **32.** $3x^{\frac{2}{3}} - \frac{x^3}{3} + C$

33. $\frac{10}{3}x^{\frac{3}{2}} - \frac{4}{3}x^{\frac{3}{4}} + C$ **34.** $x + 6x^2 + 12x^3 + C$

35. $\frac{25x^3}{3} - 15x^2 + 9x + C$ **36.** $\frac{4x^3}{3} - 49x + C$

37. $\frac{5x^3}{3} + 4x^2 - 4x + C$ **38.** $6x^3 + \frac{27x^2}{2} + 4x + C$

39. $\frac{x^3}{48} + 2x - 16x^{-1} + C$ **40.** $\frac{4x^3}{3} - 4\ln|x| - \frac{x^{-3}}{3} + C$

41. $-\frac{2}{5}x^{-\frac{5}{2}} + C$ **42.** $\frac{12}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$

43. $\frac{x^2}{2} - \frac{14}{3}x^{\frac{3}{2}} + 12x + C$ **44.** $3x^{\frac{1}{3}} + 6x^{\frac{2}{3}} + 4x + C$

45. $\frac{2x^3}{3} - x + 3x^{-1} + C$ **46.** $\ln|x| + 3x^{-1} - x^{-4} + C$

47. $7x + 11\ln|x| + C$ **48.** $\frac{x^4}{4} - \ln|x| + C$

49. $-\frac{3}{x} - \ln|x| + C$ **50.** $\ln|x| + \frac{5}{3x^3} + C$

51. $3x^2 - 31x + 5\ln|x| + C$

52. $-3\ln|x| - 17x^{-1} + 5x^{-2} + C$

53. $2x^3 + \frac{3a-2b}{2}x^2 - abx + C$

54. **(a)** $\frac{1}{2(\sqrt{\sqrt{2x}-3})(\sqrt{2x})}$

(b) $5\sqrt{\sqrt{2x}-3} + C$

55. **(a)** $-\frac{2x^3}{(x^2+4)^3}$

(b) $-\frac{x^2+2}{2(x^2+4)^2} + C$

56. **(c)** $\frac{1}{4}x^4 \ln x + \frac{4\ln 2 - 1}{16}x^4 + C$